Uncertainty Analysis

Calculating uncertainties is an essential part of producing a practical measured value. Without the uncertainty the measurement is incomplete in that there is no data to present the range within which the real value might reasonably lie.

Uncertainty then expresses the dispersion of the values associated with the particular measurement and with a given probability, whereas an error is the deviation of the measured value from the true value and can arise from random and systematic sources.

In this work, there are errors in the recording and measurement in all the stages of a test run, starting with the discharge stage, the energy supplied by the battery to the generator and then the pulse charging. These need to be individually identified by compiling the various elements that make up the value. For example, in the case of the energy supplied to the generator, we have the supply voltage, the average current delivered to the device and the run time for which it is operated. All these combine to produce a value for the energy supplied in Joules. As such the uncertainty in each of those component values must be brought together to derive the uncertainty for the energy supplied.

From statistics theory, and using a simplified method of error propagation (cf partial derivatives method), the total relative uncertainty of a value derived from the multiplication of its component values i.e. $E_{(Supplied)} = V_{(av)} \,.\, I_{(av)} \,.\, t_{(Run)}$ J, is comprised of the sum of the individual relative uncertainties:

The relative or fractional uncertainty is the ratio of the uncertainty to the measured value e.g. 0.1V/12V = 0.0083 (8.3E-3). The relative uncertainty of a calculated value, made up of several components multiplied together, is equal to the sum of the individual relative uncertainties.

The Absolute uncertainty is the resulting uncertainty for the measured value and is derived from the measured value itself and the combined associated relative uncertainty. This will be easier to see in the examples for the generator measurements below.

Energy Supplied (Δ_{Es})

Rel. $U_V = \delta_V = \Delta_V / V = 0.1/12.5 = 8.0E\text{-}3$

Rel. $U_I = \delta_I = \Delta_I / I = 0.01/0.75 = 1.3E\text{-}2$

Rel. $U_t = \delta_t = \Delta_t / t = 0.5/900 = 5.5E\text{-}4$

Rel. $U_{Es} = \delta_{Es} = \delta_V + \delta_I + \delta_t = 8.0E\text{-}3 + 1.3E\text{-}2 + 5.5E\text{-}4 = 2.16E\text{-}2$

Also $\delta_{Es} = \Delta_{Es} / E_{(Supplied)} \;\therefore\; \Delta_{Es} = \delta_{Es} \times E_{(Supplied)} = 2.16E\text{-}2 \times 12{,}200J = 263J$

Although in these two experiments, extrapolation and interpolation have been used to determine the final value of the energy supplied, and a value of Δ_{Es} of 100J (0.1kJ) could be used based on the uncertainty in plotting and reading the X axis value, since the computational value has been calculated at 263J, this larger value has been used in the calculation of the CoP uncertainty.

Energy Received (Δ_{Er})

For the energy discharged by the CBA, and subsequently returned to the receiving battery, the absolute uncertainty Δ_{Er} = 360J based on the device specifications and the uncertainty in the measured energy dissipated of 0.1Wh (360J), despite the recorded value having a resolution of 0.001Wh (3.6J)

$E_{(Received)} = E_{(Discharged)}$ (direct measurement) J

So similarly:

Rel. $U_{Er} = \delta_{Er} = \Delta_{Er} / E_{(Received)} = 360 / 41{,}000 = 8.78E\text{-}03$

Coefficient of Performance (Δ_{CoP})

For the uncertainty of the CoP, and using an example of 3.36 (derived from the energy received divided by the energy supplied), the total uncertainty of a value derived from the division of its component values i.e. CoP = $E_{(Received)} / E_{(Supplied)}$, is calculated from the sum of the component relative uncertainties such that:

$$\delta_{CoP} = (\delta_{Er} + \delta_{Es}) = \Delta_{CoP} / CoP$$

$$\therefore \Delta_{CoP} = (\delta_{Er} + \delta_{Es}) \times CoP = (8.78E\text{-}03 + 2.16E\text{-}02) \times 3.36 = 0.10$$

Therefore in this example the value of CoP is 3.36 ± 0.10, so the actual value lies in the range 3.26 - 3.46.

Available Power (Δ_{P})

For the available external power, the total relative uncertainty of a value derived from the multiplication of its component values i.e. $P_{(available)} = E_{(available)}$ / time is comprised of the sum of the individual relative uncertainties:

Rel. $U_E = \delta_E = \Delta_E / E = 0.1/54.3 = 1.8E\text{-}03$

Rel. $U_t = \delta_t = \Delta_t / t = 0.1/17 = 5.9E\text{-}03$

Rel. $U_P = \delta_P = \delta_E + \delta_t = 1.8E\text{-}03 + 5.9E\text{-}03 = 7.7E\text{-}03$

Also $\delta_P = \Delta_P / P_{(Available)} \therefore \Delta_P = \delta_P \times P_{(Available)} = 7.7E\text{-}03 \times 53.2W = 0.41W$

These calculations are built into the spreadsheet supplied in the Appendices after certain key data has been entered manually. A detailed description of all the spreadsheet cells and the calculations carried out is provided in the 'Spreadsheet Guidance Notes'.

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