Physics Letters B 767 (2017) 431–436

Contents lists available at ScienceDirect

Physics Letters B

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Vacuum radiation induced by time dependent electric field

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ARTICLE INFO

Article history:
Received 1 October 2016
Received in revised form 6 January 2017
Accepted 31 January 2017
Available online 16 February 2017
Editor: B. Grinstein

Keywords:
QED
Strong field
Radiation
Vacuum polarization

ABSTRACT

Many predictions of new phenomena given by strong field quantum electrodynamics (SFQED) will be tested on next generation multi-petawatt laser facilities in the near future. These new phenomena are basis to understand physics in extremely strong electromagnetic fields therefore have attracted wide research interest. Here we discuss a new SFQED phenomenon that is named as vacuum radiation. In vacuum radiation, a virtual electron loop obtain energy from time dependent external electric field and radiate an entangled photon pair. Features of vacuum radiation in a locally time dependent electric field including spectrum, characteristic temperature, production rate and power are given.

1. Introduction

Strong field quantum electrodynamics (SFQED) is a very active field of research recently. With progresses in next generation laser facilities such as ELI [1], HiPER [2] and XCELS [3], many predictions of fascinating phenomena made long before will be tested in the near future, such as Schwinger pair production [4–6], vacuum birefringence [7], Unruh radiation [8], photon splitting [9,10] and elastic photon scattering [11]. For recent reviews, see [12–14].

Among these predictions, Schwinger pair production [6] is of specific importance for its nonlinear, non-perturbative nature and unique phenomenon. In the simplest case of a constant electric field $\mathcal{E}$, its production rate per unit time and volume is ($\hbar = c = 1$)

$$\frac{N_{e^+e^-}}{V_4} = \frac{2e^2\mathcal{E}^2}{8\pi^2} e^{-\frac{\pi E_{cr}}{\mathcal{E}}}, \tag{1}$$

where $E_{cr} = m^2/e \approx 1.3 \times 10^{18}$ V/m is the very high critical Schwinger field above which Schwinger pair production process can numerously happen (m and e are the mass and charge of electron). To lower the requirement on laser intensity to observe Schwinger pair production, several approaches have been proposed, such as overlap a rapidly changing electromagnetic field with a strong and slowly varying electric field [15], superimpose a plane wave x-ray with a strongly optical laser [16–18], deploy a sequence of alternating-sign time dependent electric field pulses [19], collide a relativistic nucleus with a strong low frequency and a weak high-frequency laser filed [20,21] or multiple colliding electromagnetic pulses [22]. In addition, reductionism interpretation [23] and higher order processes [24] also deepen our understanding of the vacuum.

Although electron is the lightest charged particle, a positron–electron pair is not the lightest state that electromagnetic field can create from vacuum. In this letter, an external field induced vacuum radiation (VR) is discussed. In this process, a virtual electron loop obtains energy from locally time dependent electric field background and creates a pair of entangled photons. Different from VR which has a microscopic origin that will be presented below, an adiabatic mechanism that also generate photon pairs from vacuum was discussed by A. Di Piazza et al. [25]. In their research, a strong rotating magnetic field generates electron, photon and neutrino pairs from vacuum through the non static vacuum birefringence aroused by the changing field. The spectrum of photons created through that adiabatic mechanism is flat and the production rate per unit volume is proportional to e^8 when the field is weak ($B \ll B_{cr}$ where $B_{cr} = E_{cr}/c$ is the critical magnetic field). Besides that, two SFQED phenomena that generate photons, i.e., enhancement of elastic photon scattering by employing large aperture laser beams [26] and higher harmonics generation [27,28] were also discussed recently. A comparison of VR with these three phenomena will be given in the conclusion.

To see the microscopic mechanism of VR, an enlightening semi-classical analogy can give a very simple picture of how an electron

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http://dx.doi.org/10.1016/j.physletb.2017.01.076

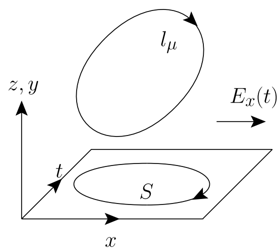


Fig. 1. Electron loop and its projection on $x-t$ plane.

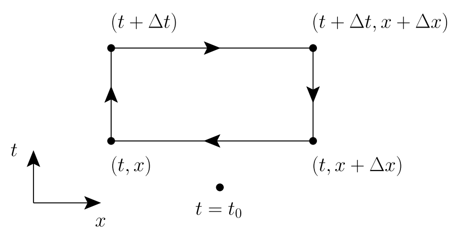


Fig. 2. ΔE obtained by a rectangular loop on $x-t$ plane.

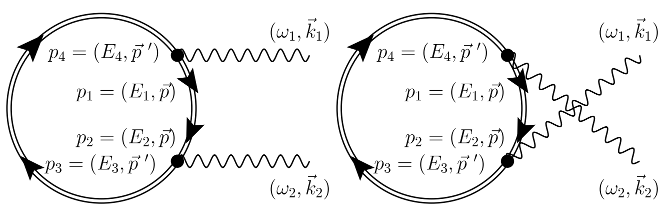


Fig. 3. Lowest order QED diagram contribute to VR, double lines are electron propagators in external field.

loop extract energy from time dependent electric field and generate photons. In classical electrodynamics, the infinitesimal energy change dE of a particle charged Q in external electric field $\mathbf{E}$ is $dE = Q\,\mathbf{E} \cdot d\mathbf{x}$. The energy obtained by this particle along a closed loop l_μ as shown in Fig. 1 is then

$$\Delta E = Q \oint d\mathbf{l} \cdot \mathbf{E}. \tag{2}$$

ΔE vanishes for a constant field, but for a time dependent electric field, i.e. the field is locally $\mathbf{E}_t = (E_{t_0} + E'_{t_0}(t-t_0), 0, 0)$, this is simply $\Delta E = Q E'_{t_0} S$, where S is the signed area enclosed by the projection of l_μ on $x-t$ plane and E'_{t_0} is the local time derivative of electric field. To demonstrate this more clearly, we estimate the ΔE for a rectangular loop on $x-t$ plane as shown in Fig. 2. The loop obtains an energy of $E_{t_0}\Delta x + E'_{t_0}(t+\Delta t - t_0)\Delta x$ along the upper side, and it lost an energy of $E_0 \Delta x + E'_{t_0}(t-t_0)\Delta x$ along the lower side. As a result, the pure energy gain is $\Delta E = \Delta t \Delta x E'_{t_0}$.

Hence, with the presence of time dependent electric field, a virtual $e^+\ e^-$ pair in the vacuum can obtain non-vanishing energy. This energy is released as a radiation when the virtual $e^+\ e^-$ pair annihilate, which is described by the diagrams in Fig. 3. Note that since VR is a process that mainly involves a virtual electron loop, it happens within a space time scale of $1/m \sim 4 \times 10^{-13}$ m. An electric field linearly growing within a region of such space time scale allows VR to happen, continuous growing to infinity is not necessary.

2. Vacuum radiation in QED

Quantitative investigation of VR needs a QED calculation, therefore we start from the general form of VR probability with the presence of an arbitrary external field F in QED, which is

$$N_F^{VR} = \frac{1}{2}\sum_{s_1,s_2}\int \frac{d\mathbf{k}_1^3}{(2\pi)^3}\int \frac{d\mathbf{k}_2^3}{(2\pi)^3}\frac{1}{(2E_{\mathbf{k}_1})(2E_{\mathbf{k}_2})}$$
$$\langle 0|\mathbf{k}_1 s_1, \mathbf{k}_2 s_2\rangle_F \langle \mathbf{k}_1 s_1, \mathbf{k}_2 s_2|0\rangle_F, \tag{3}$$

where $k_{1,2}$ and $s_{1,2}$ are momenta and spins of radiated photons, $\langle \mathbf{k}_1 s_1, \mathbf{k}_2 s_2|0\rangle_F$ is the transition amplitude of VR with the presence of external field F and the additional $1/2$ comes from double counting of final states.

For the Lagrangian of QED is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/_\mu - eA\!\!\!/_\mu + m)\psi, \tag{4}$$

the lowest order contribution to VR is from the 1-loop diagram shown in Fig. 3. Including permutation, the transition amplitude of this diagram reads

$$\langle \mathbf{k}_1 s_1, \mathbf{k}_2 s_2|0\rangle_F^{1-loop}$$
$$\approx -e^2\int d^4x_1 d^4x_2 e^{ik_1x_1}e^{ik_2x_2}$$
$$\mathrm{Tr}[G_F(x_1,x_2)\gamma^\mu G_F(x_2,x_1)\gamma^\nu]$$
$$(\epsilon_\mu^{*s_1}(\mathbf{k}_1)\epsilon_\nu^{*s_2}(\mathbf{k}_2) + \epsilon_\nu^{*s_1}(\mathbf{k}_1)\epsilon_\mu^{*s_2}(\mathbf{k}_2)), \tag{5}$$

where ϵs are the photon polarization vectors and G_F the electron propagator in external field F.

3. Electron propagator in external field

To obtain the transition amplitude given in Eq. (5), the electron propagator G_F in time dependent electric field F is needed. However, exact electron propagator in arbitrary external field is a hard problem. There are several frequently used methods about electron propagator in external field, but all these approaches have their limitations: Volkov propagator is applicable only to plane wave fields [29,30], proper time method [6] needs constant fields and worldline instanton approaches also demands specific form of external field [31]. Besides these methods, there are also some recent progresses in electron wave function and propagator in external field of general space–time structure [32,33], but their applicability are also limited by pre-requisitions such as that the initial energy of electron is the largest dynamical energy scale in the problem.

A possible solution is to give the exact electron propagator in arbitrary external field formally as a path integral [34], which is convenient to incorporate time dependent external fields. In the path integral method, the electron propagator from x_a to x_b in an arbitrary external field A^μ is

$$G_A(x_a,x_b) = -i\int_0^\infty d\tau \int \mathcal{D}\bar{z}\mathcal{D}z\mathcal{D}x\mathcal{D}q \exp[i\int_0^\tau d\omega$$
$$(e\bar{z}A\!\!\!/z - \frac{1}{2i}(\dot{\bar{z}}z - \bar{z}\dot{z}) + q\dot{x} - \bar{z}q\!\!\!/z)], \tag{6}$$

where derivatives are with respect to the parameter time ω, e.g., $\dot{x} = \partial x/\partial\omega$. This path integral should be understood as the continuous limit of

$$G_A(x_a,x_b)$$
$$= -i\int_0^\infty d\tau \exp(-i\tau m)\int \prod_{j=1}^{N+1}\frac{dq_j^4}{(2\pi)^4}\prod_{j=1}^{N}dx_j^4$$
$$\int \prod_{j=1}^{N+1}\frac{d\bar{z}_j^4 dz_j^4}{(2\pi)^4}\exp[i\sum_{j=1}^{N+1}\epsilon(e\bar{z}_jA\!\!\!/(x_j)z_j$$
$$-i\bar{z}_j\dot{z}_j + q_j\dot{x}_j - \bar{z}_jq\!\!\!/_jz_j)], \tag{7}$$

where the derivatives become differences, e.g., $\dot{x}_{\nu j} = (x_{\nu j} - x_{\nu j-1})/\epsilon$ ($\epsilon = W/(N+1)$). Additionally, the two ends of all the paths, i.e., x_0 and x_{N+1}, are fixed at x_a and x_b, respectively. In this path integral, the measure on internal space $\bar{z}, z \in C_4$ is

$$\int \mathcal{D}\bar{z}\mathcal{D}z$$

$$\equiv \exp(-i\tau m) \int_{-\infty}^{\infty} \prod_{j=1}^{n+1} \frac{id\bar{z}_j dz_j}{2\pi} \exp[-\frac{1}{2}\bar{z}z|_{\tau_a}^{\tau_b}]. \tag{8}$$

Integrate over $\mathcal{D}\bar{z}$, $\mathcal{D}z$ and redefine q_j, Eq. (7) becomes

$$G_A(x_a, x_b)$$

$$= -i \int_0^{\infty} d\tau \exp[-i\tau m] \int \mathcal{D}x^4 \mathcal{D}q^4 \prod_{j=1}^{N+1} \frac{1}{1 - i\epsilon \not{q}_j}$$

$$\exp[i \sum_{j=1}^{N+1} (q_{\mu j} + eA_\mu(x_j))(x_j^\mu - x_{j-1}^\mu)]. \tag{9}$$

4. Simplest example: single propagator loop in time dependent electric field

To apply path integral propagator in Eq. (9) to the amplitude of VR in Eq. (5), the time dependent electric field is needed. The general form of a time dependent electric field $\mathbf{E}(t)$ around t_0 is

$$\mathbf{E}_{t_0} + \mathbf{E}'_{t_0}(t - t_0) + \mathcal{O}((t - t_0)^2). \tag{10}$$

Without loss of generality, fix the electric field along x axes, then the vector potential of this field can be locally gauge fixed to

$$A^\mu(t) = (-E_{t_0}(x - x_0) - E'_{t_0}(x - x_0)(t - t_0), 0, 0, 0)$$
$$+\mathcal{O}((x - x_0)(t - t_0)^2). \tag{11}$$

Move (t_0, x_0) to the origin, ignore higher order terms which are not important for VR as we will show later, the field can be rewritten locally as

$$\mathbf{E}(t) = (E_0 + \eta t, 0, 0) + \mathcal{O}(t^2) \tag{12}$$

around the origin, where $E_0 = E_{t_0}$ and $\eta = E'_{t_0}$. Then the vector potential is locally

$$A^\mu(t) = (-E_0 x - \eta x t, 0, 0, 0) + \mathcal{O}(xt^2). \tag{13}$$

To ensure a clean background that Schwinger pair production is ignorable, we further restrict our calculation in the weak external field regime where $|E_0| \ll E_{cr}$ and assume the field varies slowly ($\eta \ll mE_{cr}$). Considering that the field strength of next generation laser facilities such as ELI is far below E_{cr} and their frequencies are several magnitudes below m, these two conditions are always fulfilled in foreseeable future.

Before apply the electron propagator in time dependent field to the amplitude of VR in Eq. (5) which consists of a two electron propagator loop, an analysis of the simplest loop that only consist one electron propagator ($x_b = x_a$) in time dependent electric field would clearly demonstrate the mechanism how an electron loop extract energy from time dependent electric field. Assume the external field is very weak ($\eta \ll mE_{cr}$ and $E_0 \ll E_{cr}$), the terms introduced by external field can be taken as small perturbations, a self closed propagator is hence (the notation agrees with [34])

$$G_F(x_a, x_a)$$

$$= -i \int_0^{\infty} d\tau \exp[-i\tau m] \int \mathcal{D}x^4 \mathcal{D}q^4$$

$$\exp[i \int_0^{\tau} d\omega q_\mu \dot{x}^\mu] \prod_{j=1}^{N+1} \frac{1}{1 - i\epsilon(\not{q}_j)}$$

$$\exp[-\frac{ie\eta\mathcal{S}}{N} \sum_{i=1}^{N} t_i + ie\eta\mathcal{O}(\delta x^3)] \exp[-ieE_0 S], \tag{14}$$

where

$$\mathcal{S} = \sum_{i,j} M_{ij} \delta t_i \delta x_j \tag{15}$$

is the signed area of the closed path $l_\mu = (x_0 \to x_1 \to x_2 ... \to x_{N+1} = x_0)$ projected on $x - t$ plane as shown in Fig. 1. In the definition of $\mathcal{S}$, $\delta t_i = t_i - t_{i-1}$, $\delta x_j = x_j - x_{j-1}$ and

$$M_{ij} = \begin{cases} 0 & i = j \\ 1/2 & i > j \\ -1/2 & i < j \end{cases}. \tag{16}$$

The external field contributes the two terms in the last line of Eq. (14), the first term which is induced by the time derivative η of external field contributes a pure vacuum current proportional to $(e\eta S, 0, 0, 0)$ and the second term is introduced by external field strength E_0. When E_0 is weak ($E_0 \ll m^2/e$), its contribution to $G_F(x_a, x_a)$ can be approximated by a factor of

$$L_{H-E}(E_0)/(\frac{1}{2}E_0^2) \approx 1 + \frac{4\alpha^2 E_0^2}{45m^4}, \tag{17}$$

where L_{H-E} is the Heisenberg–Euler Lagrangian [6]. This factor can be taken as a deformation to the loop and is neglectable when $E_0 \ll E_{cr}$. The physics reason for ignoring the influence of weak constant field strength on electron loop deformation is simple. Electron is the heaviest virtual particle involved. VR happens at the space–time scale of $1/m$, and 4-momenta of virtual particles involved are of the scale of m. Hence the deformation to the electron loop introduced by a constant field $\ll m^2/e = E_{cr}$ is weak. Consequently, the modification to the amplitude of electron loop is also weak.

From the semiclassical analogy and the first term in the last line of Eq. (14), we can see that the main features of the "vacuum current" $(e\eta\mathcal{S}, 0, 0, 0)$ induced by the local linearly growing electric field are determined by the energy absorbed by the electron loop $e\eta\mathcal{S}$. Corresponding amplitude is the part of electron loop amplitude whose signed area equals S. Then we need to change the integration variable from x_j^μ to $\mathcal{S}$. However, the calculation of corresponding Jacobian is too complicated, and we choose to deal with it in an alternative way.

We take the signed area $\mathcal{S}$ as an independent variable which has a distribution amplitude

$$\mathcal{F}_{nF}(\mathcal{S}_0)$$

$$= \frac{\langle \int \prod d^4x_i \mathrm{Tr}[G_F(x_0, x_1)\gamma^\mu ... G_F(x_n, x_0)|_{\mathcal{S}=\mathcal{S}_0}] \rangle}{\langle \int \prod d^4x_i \mathrm{Tr}[G_F(x_0, x_1)\gamma^\mu ... G_F(x_n, x_0)] \rangle}, \tag{18}$$

where the subscript "n" denotes that the loop includes n electron propagators and F the external field. As a distribution amplitude, it should conserve the probability by satisfying

$$\int dS \mathcal{F}_{nF}(\mathcal{S})\mathcal{F}^*_{nF}(\mathcal{S}) = 1. \tag{19}$$

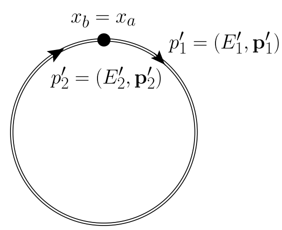


Fig. 4. Simplest loop that only consist one electron propagator, the double line is an electron propagator in external field.

When both $E_0 \ll E_{cr}$ and $\eta \ll mE_{cr}$ are satisfied, external field induced terms in Eq. (14) are small perturbations and the distribution amplitude can be approximated by its vanishing external field limit $\mathcal{F}_{n0}(\mathcal{S})$. Applying this to Eq. (14), the single propagator loop becomes

$$G_F(x_a, x_a) \approx \int \frac{dE'_1 d\mathbf{p}'^3}{(2\pi)^4} d\mathcal{S}\mathcal{F}_{10}(\mathcal{S}) \int_0^\infty d\tau$$
$$e^{-im\tau} e^{ix_a(p'_1 - p'_2)} \exp[i\tau \frac{\not p'_1 + \not p'_2}{2}], \tag{20}$$

where $p'_1 = (E'_1, \mathbf{p}')$, $p'_2 = (E'_2, \mathbf{p}') = (E'_1 + e\eta\mathcal{S}, \mathbf{p}')$ are momenta at the two ends of the propagator as shown in Fig. 4 (i.e., q_1 and q_{N+1} in the path integral). When the field vanishes, this approximate propagator degenerates to the correct form of electron propagator in vacuum [34].

5. Vacuum radiation

Then we apply the electron propagator in Eq. (9) to the lowest order transition amplitude of VR given by Eq. (5). In this amplitude, the closed loop composed of two electron propagators has

$$\int dx_b^4 G_\eta(x_a, x_b)\gamma^\mu G_\eta(x_b, x_a)\gamma^\nu$$
$$\approx \int \frac{dE_1 d\mathbf{p}^3}{(2\pi)^4} \frac{dE_3 d\mathbf{p}'^3}{(2\pi)^4} d\mathcal{S}\mathcal{F}_{20}(\mathcal{S}) \int_0^\infty d\tau_1 d\tau_2 e^{-im(\tau_1+\tau_2)}$$
$$\int d^4x_b e^{ix_a(p_1-p_4)} e^{ix_b(p_3-p_2)} \exp[i\tau_1 \frac{\not p_1 + \not p_2}{2}]\gamma^\mu$$
$$\exp[i\tau_2 \frac{\not p_3 + \not p_4}{2}]\gamma^\nu, \tag{21}$$

where $p_1 = (E_1, \mathbf{p})$, $p_2 = (E_1 + \frac{e\eta\mathcal{S}}{2}, \mathbf{p})$, $p_3 = (E_3, \mathbf{p}')$, $p_4 = (E_3 + \frac{e\eta\mathcal{S}}{2}, \mathbf{p}')$ are momenta at the ends of propagators as shown in Fig. 3 and $\mathcal{F}_{20}(\mathcal{S})$ the distribution amplitude of $\mathcal{S}$ defined in Eq. (18). Similar to the single electron propagator loop case, the time derivative η contributes a pure vacuum current proportional to $(e\eta S, 0, 0, 0)$ and the effect of external field strength $E_0 \ll E_{cr}$ on loop deformation is neglectable. The VR amplitude can then be simplified to

$$\langle \mathbf{k}_1 s_1, \mathbf{k}_2 s_2 | 0 \rangle_\eta^{1-loop}$$
$$\approx -\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \int d\mathcal{S}\mathcal{F}_{20}(\mathcal{S})$$
$$(\epsilon_\mu^{*s_1}(\mathbf{k}_1)\epsilon_\nu^{*s_2}(\mathbf{k}_2)\Pi_2^{\mu\nu}(l) + \epsilon_\nu^{*s_1}(\mathbf{k}_1)\epsilon_\mu^{*s_2}(\mathbf{k}_2)\Pi_2^{\mu\nu}(-l))$$
$$\delta(\omega_1 + \omega_2 - e\eta S), \tag{22}$$

in which $l = (0, \mathbf{k}_1)$ and

$$\Pi_2^{\mu\nu}(l) = -\int \mathrm{Tr}[\gamma^\mu \frac{d^4p}{\not p - m + i\epsilon}\gamma^\nu \frac{e^2}{\not p + \not l - m + i\epsilon}], \tag{23}$$

is the QED vacuum polarization amplitude to the e^2 order. The δ functions in Eq. (22) indicate that created two photons have the same energy and opposite momentum. It is not a surprise that the amplitude of VR in weak external field limit is proportional to vacuum polarization amplitude for they are the same diagram except the former has a time dependent electric field background while the later has not.

In QED, the vacuum polarization amplitude

$$\Pi^{\mu\nu}(l) = (l^2 g^{\mu\nu} - l^\mu l^\nu)\Pi(l^2), \tag{24}$$

where $(l^2 g^{\mu\nu} - l^\mu l^\nu)$ is its transverse projection tensor structure and $\Pi(l^2)$ is logarithmical ultraviolet divergent. $\Pi(l^2)$ itself also has two parts and can be written as

$$\Pi(l^2) = \Pi(0) + \hat{\Pi}(l^2), \tag{25}$$

where $\Pi(0)$ is the infinite, energy scale independent part of vacuum polarization that absorbed into the definition of physical photon therefore has no observable effect, and

$$\hat{\Pi}(l^2) = -\frac{2\alpha}{\pi}\int_0^1 dx x(1-x)\ln(\frac{m^2}{m^2 - x(1-x)l^2}) + \mathcal{O}(\alpha^2),$$

is the renormalized vacuum polarization that is energy scale dependent and responsible for all observable effects. When $l^2/m^2 \ll 1$, the renormalized vacuum polarization has

$$\hat{\Pi}(l^2) \approx -\frac{\alpha}{15\pi}\frac{l^2}{m^2} + \mathcal{O}\big((\frac{\alpha l^2}{m^2})^2\big) \tag{26}$$

As a result, according to renormalization theory of QED, the physical amplitude of VR is

$$\langle \mathbf{k}_1 s_1, \mathbf{k}_2 s_2 | 0 \rangle_\eta^{1-loop}$$
$$\approx -\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \int d\mathcal{S}\mathcal{F}_{20}(\mathcal{S})$$
$$(\epsilon_\mu^{*s_1}(\mathbf{k}_1)\epsilon_\nu^{*s_2}(\mathbf{k}_2) + \epsilon_\nu^{*s_1}(\mathbf{k}_1)\epsilon_\mu^{*s_2}(\mathbf{k}_2))$$
$$\delta(\omega_1 + \omega_2 - e\eta S)\hat{\Pi}_2^{\mu\nu}(l). \tag{27}$$

The production rate of VR photon pairs per unit time and volume in a time dependent electric field is then

$$\frac{d^4N^{VR}}{dx^4} \approx \frac{1}{2}\int \frac{d\mathbf{k}_1^3}{(2\omega_1)^2} \frac{2(g_{\mu\alpha}g_{\nu\beta} + g_{\nu\alpha}g_{\mu\beta})}{(2\pi)^6}$$
$$\Pi_2^{\alpha\beta}\big((0, \mathbf{k}_1)\big)\Pi_2^{\mu\nu}\big((0, \mathbf{k}_1)\big)F(\omega_1)$$
$$= \frac{1}{2(2\pi)^6}\int \frac{d\mathbf{k}_1^3}{(2\omega_1)^2} 12\mathbf{k}_1^4 \hat{\Pi}_2^2(-\mathbf{k}_1^2)F(\omega_1), \tag{28}$$

where

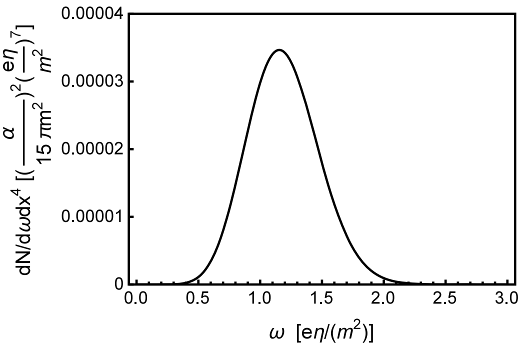
$$F(\omega_1) = \frac{2}{e\eta}\mathcal{F}_{20}^2(\frac{2\omega_1}{e\eta}) \tag{29}$$

is the distribution function deduced from the distribution amplitude $\mathcal{F}_{20}(\mathcal{S})$.

Apply the approximate distribution amplitude $\mathcal{F}_{20}(\mathcal{S})$ obtained in the supplemental material, the spectrum of created photons can be extracted from Eq. (28), which gives

$$\frac{dN_\gamma}{d\omega dx^4} \approx \frac{3}{16\pi^5}(\frac{\alpha}{15\pi})^2 \frac{\omega^8}{m^4} F(\omega) \tag{30}$$

when $\eta \ll m^3/e$ is satisfied. This spectrum is shown in the top panel of Fig. 5, which has a peak at $\omega_{peak} = 2e\eta/\sqrt{3}m^2$ that can be



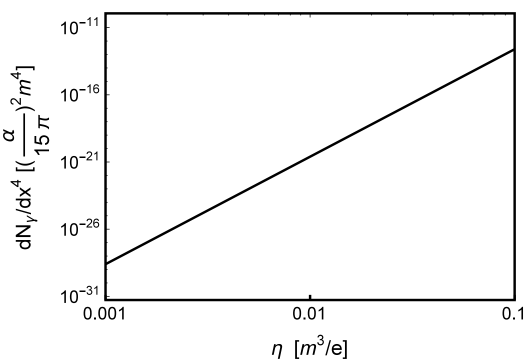


Fig. 5. Spectrum ($dN_\gamma/d\omega dx^4$) and production rate (dN_γ/dx^4) of vacuum radiation, normalized by $(\frac{\alpha}{15\pi m^2})^2(\frac{e\eta}{m^2})^7$ and $(\frac{\alpha m^2}{15\pi})^2$.

taken as its characteristic temperature. Corresponding production rate is

$$\frac{dN_\gamma}{dx^4} \approx \frac{35}{4608\pi^5}\left(\frac{\alpha}{15\pi}\right)^2 \frac{(\frac{e\eta}{m^2})^8}{m^4}, \tag{31}$$

which is also shown in Fig. 5, and the power is

$$\frac{dP}{dx^3} \approx \frac{1}{18\pi^5\sqrt{3\pi}}\left(\frac{\alpha}{15\pi}\right)^2 \frac{(\frac{e\eta}{m^2})^9}{m^4}. \tag{32}$$

An interesting feature of the generated photon pair in VR is the entangled spin. Apply $\Pi^{\mu\nu}$ in Eq. (24) to the VR transition amplitude in Eq. (22), the transition amplitude of VR

$$\begin{aligned}\langle \mathbf{k}_1 s_1, \mathbf{k}_2 s_2 | 0 \rangle_\eta^{1-loop} &\propto \Pi^{\mu\nu}(\vec{\mathbf{k}}_1)\epsilon_\mu^{*s_1}(\mathbf{k}_1)\epsilon_\nu^{*s_2}(-\mathbf{k}_1) \\ &\propto \epsilon^{*s_1}(\mathbf{k}_1)\cdot\epsilon^{*s_2}(-\mathbf{k}_1),\end{aligned} \tag{33}$$

is proportional to the product of photon polarization vectors, therefore the two photons have entangled spin [35]. This entanglement is demanded by angular moment and CP conservation of electromagnetic interaction, which means the photon pair inherits the quantum number of vacuum ($J^{PC} = 0^{++}$).

6. Discussions and conclusion

There are several SFQED phenomena that radiate photons without presence of charged particles, such as photon splitting in strong magnetic field [9] or laser [10], harmonics generation [27], enhanced elastic photon scattering [11,26] and the adiabatic mechanism by A. Di Piazza [25]. VR is different from them for it is an effect of field derivatives within Compton length, while all the other 4 phenomena mentioned above are not since they are deduced from Heisenberg–Euler Lagrangian, which excludes effects of field derivatives within Compton length [6].

Besides the origin, they are also different phenomenologically. VR radiates in all directions while photon splitting is in the direction of split photon; VR has a continuous spectrum while harmonics generation and enhanced elastic photon scattering does not. Compared to the flat spectrum of photon generation in [25], VR has a characteristic temperature at $\omega_{peak} = 2e\eta/\sqrt{3}m^2$.

VR is also unique for its infrared spectrum. Its characteristic temperature $\omega_{peak} = 2e\eta/\sqrt{3}m^2$ is at the order of $\sim \eta/E_S$. Considering the fastest field variation in future laboratory will be achieved by overlapping of lasers, the highest field variation rate would be at the magnitude of ωE where ω is the laser frequency and E is the maximal field strength. Hence the characteristic temperature of VR $\sim \omega E/E_S$. Since E would be at least two magnitudes lower than E_S in the foreseeable future [1], VR temperature would also be at least two magnitudes below ω in foreseeable future. This is quite different from the 4 phenomena mentioned above.

In conclusion, the mechanism of external time dependent electric field induced vacuum radiation is discussed and this new SFQED phenomenon is explored with QED and the calculations are done with a path integral method. Features of VR, such as characteristic temperature, power, production rate and spin are given.

Acknowledgements

The authors would like to thank Dr. Antonino Di Piazza, Dr. Benjamin King, Prof. Bai-fei Shen, Prof. Xin-heng Guo, Prof. Bai-song Xie, Dr. Kazunori Itakura, Dr. Huayu Hu, Dr. Keita Seto and Dr. Zi-Liang Li for very helpful discussions. This work is supported by the Foundation of Science and Technology on Plasma Physics Laboratory, China (Grant No. 9140C680303140C68288), National Natural Science Foundation of China (Grant No. 11305157) and The Science Challenge Program.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.physletb.2017.01.076.

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